LAB 4

Image Rejection

Prerequisite: Lab 2 – Amplitude Modulation

4.1 Objective

This laboratory exercise illustrates the image problem in superheterodyne receivers. Image rejection is carried out using complex filtering. This lab introduces a processing technique that is straightforward in a software defined radio, but is virtually unavailable in a conventional hardware-based radio.

4.2 Background

Frequency Conversion

As we saw in Lab 2, most communication receivers convert a received signal at carrier frequency f_c to a signal at "intermediate" frequency f_{IF} for amplification and filtering prior to demodulation. In the USRP, the frequency conversion can be carried out by offsetting the receiver's carrier frequency from the carrier frequency of the transmitted signal. To avoid confusion, the receiver's carrier oscillator is usually referred to as a "local" oscillator, and its frequency as the "local oscillator frequency" f_{LO}. In Lab 2 we set the transmitter's carrier frequency to $f_c = 915.1\text{ MHz}$ and the receiver's local oscillator frequency to $f_{LO} = 915\text{ MHz}$. These settings provided an intermediate frequency $f_{IF} = f_c - f_{LO} = 100\text{ kHz}$.

In the USRP receiver, frequency conversion is carried out in hardware by multiplying the received signal by $\cos\left(2\pi f_{LO}t\right)$ and by $-\sin\left(2\pi f_{LO}t\right)$. For example, suppose the received signal is the AM waveform

$$r(t) = A_r\left[1+\mu\frac{m(t)}{m_p}\right]\cos\left(2\pi f_c t+\theta\right). \tag{1}$$

The receiver forms

$$\begin{aligned} r(t)\cos(2\pi f_{LO}t) &= A_r\left[1+\mu\frac{m(t)}{m_p}\right]\cos(2\pi f_c t+\theta)\cos(2\pi f_{LO}t) \\ &= \frac{1}{2}A_r\left[1+\mu\frac{m(t)}{m_p}\right]\cos\left[2\pi(f_c - f_{LO})t+\theta\right] \\ &\quad+\frac{1}{2}A_r\left[1+\mu\frac{m(t)}{m_p}\right]\cos\left[2\pi(f_c + f_{LO})t+\theta\right]. \end{aligned} \tag{2}$$

Of the two terms in Eq. (2), the first is at the intermediate frequency $f_{IF} = f_c - f_{LO}$, while the second is at a much higher frequency $f_c + f_{LO}$. The higher-frequency term is removed by filtering in the USRP, providing the "in-phase" signal

$$r_I(t) = A_r\left[1+\mu\frac{m(t)}{m_p}\right]\cos(2\pi f_{IF}t+\theta). \tag{3}$$

The receiver also forms a second signal,

$$\begin{aligned} -r(t)\sin(2\pi f_{LO}t) &= -A_r\left[1+\mu\frac{m(t)}{m_p}\right]\cos(2\pi f_c t+\theta)\sin(2\pi f_{LO}t) \\ &= \frac{1}{2}A_r\left[1+\mu\frac{m(t)}{m_p}\right]\sin\left[2\pi(f_c - f_{LO})t+\theta\right] \\ &\quad-\frac{1}{2}A_r\left[1+\mu\frac{m(t)}{m_p}\right]\sin\left[2\pi(f_c + f_{LO})t+\theta\right]. \end{aligned} \tag{4}$$

Again, the high-frequency term is removed, and the receiver provides the "quadrature" signal

$$r_Q(t) = A_r\left[1+\mu\frac{m(t)}{m_p}\right]\sin(2\pi f_{IF}t+\theta). \tag{5}$$

A conventional hardware-based receiver normally works with the in-phase signal given by Eq. (3). The USRP combines the in-phase and quadrature signals to form the complex IF signal $\tilde{r}(t)$ given by

$$\begin{aligned} \tilde{r}(t) &= r_I(t) + jr_Q(t) \\ &= A_r\left[1+\mu\frac{m(t)}{m_p}\right]\left[\cos(2\pi f_{IF}t+\theta)+j\sin(2\pi f_{IF}t+\theta)\right] \\ &= A_r\left[1+\mu\frac{m(t)}{m_p}\right]e^{j(2\pi f_{IF}t+\theta)}. \end{aligned} \tag{6}$$

This complex signal is what is actually provided to the user by *Fetch Rx Data*.

In the frequency domain, the spectrum $R(f)$ of the received signal $r(t)$ given in Eq. (1) is shown in Figure 1.

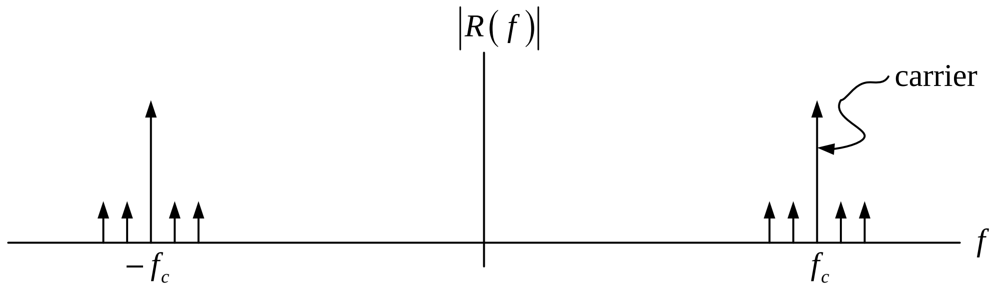

Figure 1. Spectrum of Received AM Signal

Since $r(t)$ is a real-valued signal, its spectrum contains both positive and negative-frequency components. After frequency conversion, the complex IF signal $\tilde{r}(t)$ has the spectrum $\tilde{R}(f)$ shown in Figure 2.

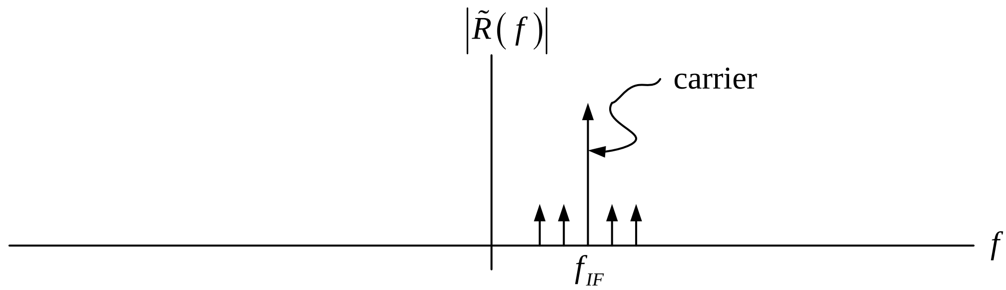

Figure 2. Spectrum of Complex IF Signal

Notice that $\tilde{r}(t)$ contains only positive frequency components.

In Lab 2 we passed the complex IF signal through a bandpass filter. Figure 3 shows the frequency response of the bandpass filter. It is intended that signals at carrier frequencies other than f_{IF} will be rejected by the filter.

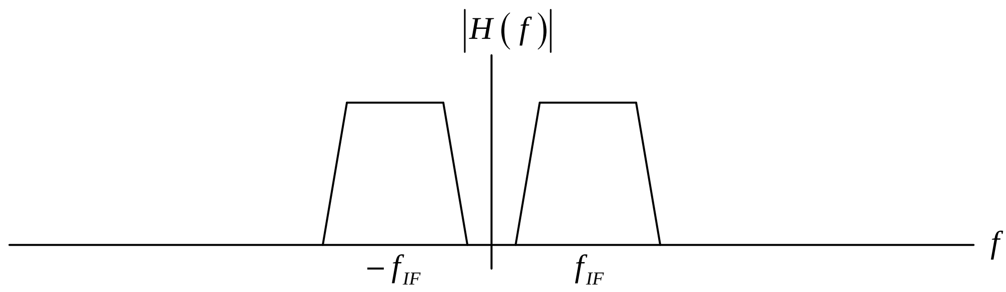

Figure 3. Frequency Response of Intermediate-Frequency Filter

Image Signal

Suppose that there is a second signal received along with the signal of Eq. (1), and that this signal is given by

$$r_2(t) = A_{r2}\left[1 + \mu_2 \frac{m_2(t)}{m_{2p}}\right]\cos\left(2\pi f_{IM} t + \theta_2\right), \tag{7}$$

where the carrier frequency f_{IM} happens to be given by

$$f_{IM} = f_{LO} - f_{IF}. \tag{8}$$

If we carry out the analysis of Eqs. (7) and (8), we find that this second signal produces the complex IF signal $\tilde{r}_2(t)$ given by

$$\tilde{r}(t) = A_{r2}\left[1 + \mu_2 \frac{m_2(t)}{m_{2p}}\right] e^{j(-2\pi f_{IF} t + \theta_2)}. \tag{9}$$

The spectrum $\tilde{R}_2(f)$ of this signal is shown in Figure 4.

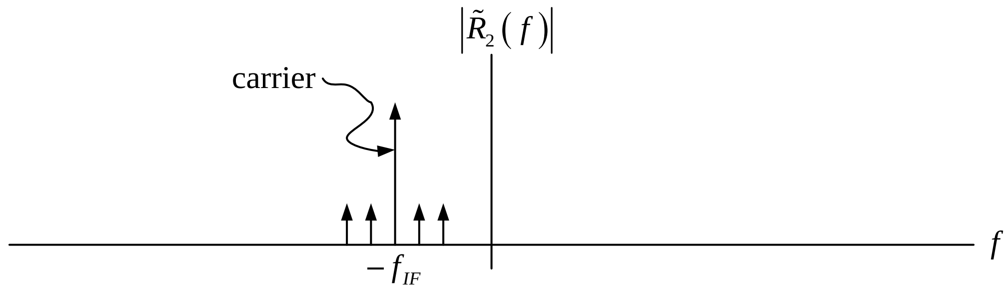

Figure 4. Spectrum of the Image Signal

The signals $r(t)$ and $r_2(t)$ are said to be "images" of one another. A glance at the frequency response shown in Fig. 3 shows that both $r(t)$ and $r_2(t)$ will pass through the IF filter, and the two signals will interfere with each other in the demodulator that follows the IF filter. The relationship between the frequencies of the two image signals is worth noting. One signal, $r(t)$, is at a carrier frequency of $f_c = f_{LO} + f_{IF}$, while the other signal, $r_2(t)$, is at a carrier frequency of $f_{IM} = f_{LO} - f_{IF}$. These carrier frequencies are symmetrically arranged about the receiver's frequency f_{LO}, the way a physical object and its image are symmetrically distant from the surface of a mirror.

Image Rejection

In a conventional receiver, the unwanted image is normally removed before the frequency conversion step. This requires placing an analog filter in the receiver front end. Clearly, we will not have the option of adding analog hardware to the USRP, so we will take advantage of some clever signal processing to remove the unwanted image signal after frequency conversion. Notice in Figure 2 that the desired signal shows up after frequency conversion at the intermediate frequency f_{IF}, while in Figure 4 we see that the unwanted image signal shows up at the intermediate frequency $-f_{IF}$. A conventional bandpass filter passes both of these signals, but if we can construct a filter with a positive-frequency-only passband, we will be able to pass the desired signal while rejecting the unwanted image. Figure 5 shows the filter frequency response we have in mind.

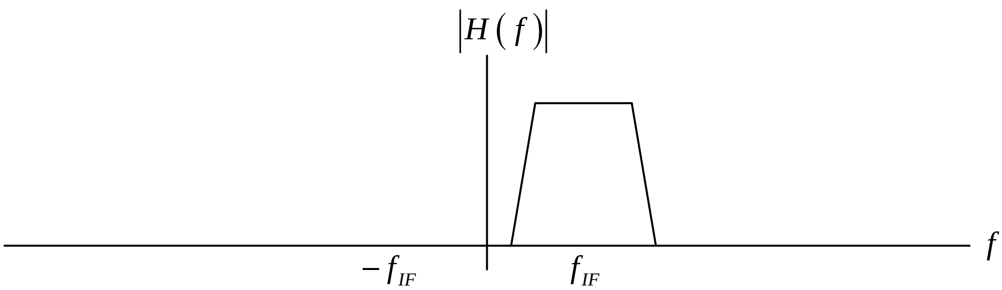

Figure 5. Frequency Response of a Complex Image-Rejection Filter

A frequency response such as that shown in Figure 5 lacks symmetry about the vertical axis, which implies that the impulse response of the filter is complex-valued. A filter with a complex-valued impulse response is difficult to produce in hardware, but, as we shall see, is easily produced in software.

On a practical note, an analog image rejection filter must be tunable if the receiver is to be capable of receiving signals at a range of carrier frequencies f_c. To make the filter tunable, it usually must be kept simple, which constrains the order of the filter to be low. Second-order filters are common in this application. A low-order filter cannot do a very thorough job of rejecting signals at the unwanted image frequency. In contrast, the complex image rejection filter of Figure 5 is centered at a fixed intermediate frequency, and does not have to be tunable. The quality of image rejection is limited only by the ability of the filter to reject signals at negative frequencies.

4.3 Pre-Lab

A complex filter has been provided in ChebyshevHilbert.gvi. Your task is to create a program to find the frequency response of this filter. There are several ways in which this can be done, and the method is left up to you.

1. Set the filter up with

   - a sampling frequency of 1MS/s
   - a high cutoff frequency of 105 kHz
   - a low cutoff frequency of 95 kHz
   - order 5
   - you can accept the default ripple value of 0.1 dB.
   - Use a *Waveform Properties* function to extract the "Y" and "dt" components of the waveform.

2. Plot the magnitude of the frequency response in decibels over the frequency range $-200\text{ kHz to }200\text{ kHz}$. (The frequency axis must be a linear scale; this is not a Bode plot.)

Note:

The input to the filter is an array of complex numbers, as is the output. If you choose to measure the frequency response by inputting sinusoidal signals at a range of frequencies, you must use complex sinusoids $e^{j2\pi ft}$ rather than conventional sinusoids $\cos(2\pi ft)$ so that negative frequencies can be distinguished from positive frequencies.

4.4 Lab Procedure

1. Connect a loopback cable and attenuator between the TX 1 and RX 2 connectors. Connect the USRP to your computer and plug in the power to the USRP. .

2. Open the AM transmitter and receiver functions that you created for Lab 2.

   Ensure that the transmitter is set up to use

   Carrier Frequency: 915.1 MHz

   IQ Rate: 500 kHz

   Gain: Not critical. 0 dB

   Active Antenna: TX1

   Message Length: 200,000 samples gives a good block of data to work with.

   Modulation Index: Start with 1.0.

   Start Frequency, Delta Frequency, Number of Tones: Not critical, but keep the highest frequency below 5 kHz. Three tones seems to work well.

   Ensure that the receiver is set up to use

   Carrier Frequency: 915 MHz

   IQ Rate: 1 MHz

   Gain: Not critical. 0 dB

   Active Antenna: RX2

   Number of Samples: Same value as the transmitted message length.

   Run the transmitter and receiver and verify that the demodulated message appears at the receiver output.

3. Modify the receiver functions by adding components to indicate the strength of the received carrier. To do this, recall that the output of an AM demodulator is

$$r_o(t) = D\left[1 + \mu \frac{m(t)}{m_p}\right]. \tag{10}$$

   Since the message $m(t)$ has an average value of zero, the average of $r_o(t)$ will be the received carrier D. You can average $r_o(t)$ by using a lowpass filter whose cutoff frequency is below the lowest frequency component of $m(t)$, or by finding a suitable signal-averaging function.

Run the transmitter and receiver and record the value of the received carrier D.

4. *Power Spectrum*

   Add the *FFT Power Spectrum and PSD* function (Analysis>>Signal Processing>> Measurement) to your receiver. Obtain the "time signal" input from the waveform produced by *Fetch Rx Data*. Attach a Boolean constant set to True to the "dB On" input. Attach a waveform graph to the "Power Spectrum/PSD" output. Change the label on the horizontal axis of the waveform graph to "Frequency."

   Run the transmitter and the receiver. Take a screenshot of the spectrum of the received signal. The spectrum should correspond to the one shown in Figure 2.

5. Use Eq. (8) to determine the image frequency f_{IM}. Set the frequency of the transmitter to f_{IM}. Run the transmitter and receiver and record the value of the received carrier D_2. Compute the *image rejection ratio* (IRR) given by

$$\mathrm{IRR} = 20\log_{10}\frac{D}{D_2}. \tag{11}$$

   (You should get a result near zero dB.)

   Take another screenshot of the spectrum of the received image signal. This time, the spectrum should correspond to the one shown in Figure 4.

6. Replace the bandpass filter in your receiver with ChebyshevHilbert.gvi. Set the transmitter's carrier frequency to $f_c = 915.1\ \mathrm{MHz}$, run the transmitter and receiver, and measure D. Now set the transmitter's carrier frequency to f_{IM}, run the transmitter and receiver, and measure D_2. Calculate the IRR and compare with the result you obtained in Step 5.

7. Save your modified receiver in a file whose name includes the letters "AMImageRx" and your initials (e.g. *AMImageRx_BAB.gvi*).

8. *Challenge Question* (Familiarity with the Hilbert transform is assumed.)

   Suppose in Step 6 you want to receive the signal at carrier frequency f_{IM} rather than the one at carrier frequency f_c. Modify *ChebyshevHilbert.gvi* to make this happen. (Hint: A single sign change in an appropriate place in the MathScript block is all that is required.)

4.5 Report

Prelab

Hand in documentation for the program you created to measure the frequency response of *ChebyshevHilbert.gvi*. Hand in documentation for the modified receiver. Also include documentation for any additional sub-functions you may have created. To obtain documentation, print out legible screenshots of the front panel and block diagram.

Submit the frequency response plot from Step 2 of the Prelab.

Lab

Submit the functions you created to measure the frequency response of *ChebyshevHilbert.gvi*. Submit the functions for the modified receiver. Also submit any additional sub-functions you may have created. Be sure your files adhere to the naming convention described in the instructions above.

Resubmit documentation for any functions you modified during the lab.

Submit the spectrum graphs required in Steps 4 and 5.

Submit your image rejection ratio computations and results from Steps 5 and 6.

If you completed the challenge question (Step 8), show how you modified *ChebyshevHilbert.gvi* and describe how you verified the result.