LAB 10

The Eye Diagram

Prerequisite: Lab 9 – Binary Phase-Shift Keying (BPSK)

10.1 Objective

Although we have displayed an eye diagram in each of the digital labs so far, we have relied on the Modulation Toolkit to produce the display. In this lab project we will learn how to create an eye diagram from scratch. We will examine how the eye diagram changes when intersymbol interference (ISI) is present in the communication channel, and we will learn how to make quantitative measurements of the amount of ISI from the eye diagram.

In this lab project you will use the BPSK transmitter and receiver that you created for Lab 9 as a "test bed" system. To create ISI, a function called *Channel.gvi* has been provided. If a second USRP is available, however, you can try transmitting from one USRP to the other using antennas to see how much ISI is actually present owing to multipath propagation in your own physical environment.

10.2 Background

Viewed as a complex baseband signal, the waveform generated by the BPSK transmitter can be written

$$\tilde{x}(t) = A\sum_{n=-\infty}^{\infty} a_n g_{TX}(t-nT), \tag{1}$$

where A is a constant that sets the transmitted average power, $a_n = \pm 1$ are the symbol values carrying the transmitted information, $g_{TX}(t)$ is the transmitted pulse shape, and T is the time between symbols. The tilde over the x signifies that this is the baseband signal; the actual transmitted signal is

$$\begin{aligned} x(t) &= \mathrm{Re}\left[\tilde{x}(t)e^{j2\pi f_c t}\right] \\ &= A\sum_{n=-\infty}^{\infty} a_n g_{TX}(t-nT)\cos(2\pi f_c t). \end{aligned} \tag{2}$$

Note that the pulse $g_{TX}(t)$ is assumed to be real valued.[7]

The transmitted signal passes through the communication channel and then is demodulated by the receiver. Let us model the communication channel as a linear time-invariant filter having baseband impulse response $h(t)$. When the signal arrives at the receiver and is demodulated, it is passed through the receiver filter having impulse response $g_{RX}(t)$. Let us designate the impulse response of the cascade of the transmitter filter, the channel, and the receiver filter as $g(t)$. That is,

$$g(t) = g_{TX}(t) * h(t) * g_{RX}(t). \tag{3}$$

The cascade of $g_{TX}(t)$ and $g_{RX}(t)$ is usually designed to be free of ISI. In this lab project we will use a cascade in which the combined filter has a raised-cosine spectral shape for that purpose. Unfortunately, the channel response $h(t)$ is not under the designer's control, and may contribute ISI to the received signal. After filtering at the receiver, the received signal can be written

$$\tilde{y}(t) = D \sum_{n=-\infty}^{\infty} a_k g(t - nT) + n(t), \tag{4}$$

where D is an amplitude constant, $g(t)$ is the pulse shape given by Eq. (3), and $n(t)$ is white Gaussian noise that has been passed through the receiver filter.

Once the received signal has been filtered, it is sampled, at the rate of one sample every T seconds. Assuming that pulse synchronization has been applied, so that the samples are taken at the correct times, the sampled signal is

$$\tilde{y}(kT) = D \sum_{n=-\infty}^{\infty} a_n g\left[(k-n)T\right] + n(kT). \tag{5}$$

These samples are compared with a threshold of zero to decide whether each received symbol is most likely to represent a $+1$ or a -1. Let us rewrite Eq. (5) as

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[7] The signal $\tilde{x}(t)$ and the impulse responses $g_{TX}(t)$ and $g_{RX}(t)$ are all actually implemented in discrete time. Writing these as continuous time functions makes the discussion much easier to read, however.

$$\tilde{y}(kT) = Da_k g(0) + D\sum_{\substack{n=-\infty \\ n\neq k}}^{\infty} a_n g\left[(k-n)T\right] + n(kT). \tag{6}$$

The first term in Eq. (6) represents the desired sample at time $t = kT$, the second term represents ISI, and the third term represents noise.

To get a sense of how much ISI a given $g(t)$ might produce in the worst case, let us suppose that $a_k = 1$ and $a_n, n \neq k$, forms a pattern that makes every ISI term negative. That is, suppose the data pattern is

$$a_n = \begin{cases} -1, & g\left[(k-n)T\right] \geq 0 \\ 1, & g\left[(k-n)T\right] < 0. \end{cases} \tag{7}$$

Then Eq. (6) becomes

$$\begin{aligned} \tilde{y}(kT) &= Dg(0) - D\sum_{\substack{n=-\infty \\ n\neq k}}^{\infty} \left|g\left[(k-n)T\right]\right| + n(kT) \\ &= Dg(0) - D\sum_{n\neq 0} \left|g(nT)\right| + n(kT). \end{aligned} \tag{8}$$

Ignoring noise, the sample given by Eq. (8) represents a "worst case" value. We write

$$V_w = Dg(0) - D\sum_{n\neq 0} \left|g(nT)\right|. \tag{9}$$

We can also identify a "best case" sample value. If every ISI term enhances the desired sample we have

$$V_b = Dg(0) + D\sum_{n\neq 0} \left|g(nT)\right|. \tag{10}$$

Traditionally, the quantity of ISI present at the output of the receiver filter is measured by either of two parameters. The *peak distortion* is defined as

$$\text{peak distortion} = \frac{D\sum_{n\neq 0}\left|g(nT)\right|}{Dg(0)}. \tag{11}$$

Alternatively, the *eye opening* is defined as

$$\text{eye opening} = 1 - \text{peak distortion}. \tag{12}$$

An example of an eye diagram is shown in Figure 1.

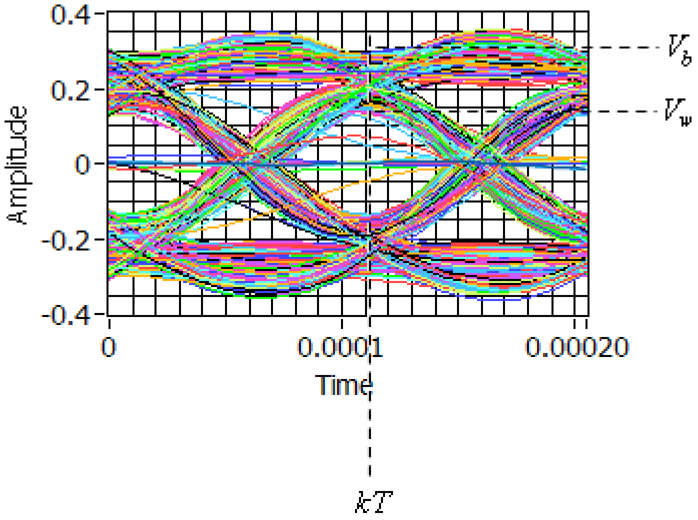

Figure 1. Eye Diagram

The sampling time kT and the best-case and worst-case voltages, V_b and V_w respectively, are shown. Note that V_b and V_w must both be measured at the same time value, kT. As you can see from the figure, the quantities $D\sum_{n\neq 0}\left|g(nT)\right|$ and $Dg(0)$ needed to find the peak distortion or eye opening are not readily apparent. If we rearrange Eqs. (11) and (12), however, we can show that

$$\text{peak distortion} = \frac{V_b - V_w}{V_b + V_w} \tag{13}$$

and

$$\text{eye opening} = \frac{2V_w}{V_b + V_w}. \tag{14}$$

Thus, for example, we can measure V_b and V_w from the eye diagram, and use Eq. (14) to calculate the eye opening. Figure 1 shows $V_b = 0.296$ and $V_w = 0.124$, giving an eye opening of 0.59, or about 60%.

Intersymbol interference is of concern because it degrades the performance of the communication system. For a BPSK system using a matched receiver filter with additive white Gaussian noise and no ISI, the probability of error is given by

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \tag{15}$$

where E_b is the average energy per bit at the receiver, $N_0/2$ is the power spectrum of the noise at the receiver input, and Q is the Gaussian probability integral. If ISI is present, the probability of error is dominated by the worst-case situation. For an eye opening η the worst-case probability of error is given by

$$P_e = Q\left(\eta\sqrt{\frac{2E_b}{N_0}}\right), \tag{16}$$

where E_b is the energy per bit without including ISI, as in Eq. (15).

10.3 Pre-Lab

Eye Diagram

1. Modify your BPSK receiver to include your own eye diagram functionality. Here is how:

   The required eye diagram is a plot of the aligned waveform at the output of *PulseAlign(real)*. The horizontal (time) axis has a duration of two symbol times. The plots corresponding to successive pairs of symbols are overlapped. To make such a plot, extract the Y-array from the aligned waveform. Also extract dt; you will need it as described below. Then use the *Reshape Array function* from the Data Types→Array subpalette to form a two-dimensional array for which each row contains two symbols of data. In the *Reshape Array function*, the first "dimension size" input is the number of rows and the second "dimension size" input is the number of columns. The number of rows is the number of curves to plot, and the number of columns is the number of samples in two symbols of data. You will need to calculate these numbers in your program. The output of *Reshape Array function* is a two dimensional array, which must be connected to the "2D DBL to 1D Cluster" function before it is ready for plotting. Use your value of dt, and the output of *Reshape Array function*. Connect the resulting cluster to a *Waveform Graph indicator*. When the *Waveform Graph indicator* is given a two dimensional array to plot, it plots each row of the array as a separate curve. This is exactly the behavior that you want to produce the eye diagram.

2. Save your modified BPSK receiver in a file whose name includes the letters "BPSKEyeRx" and your initials (e.g. *BPSKEyeRx_BAB.gvi*).

Channel Model

To test your eye diagram, you will need to introduce some ISI. *Channel.gvi*, in the *BasicUSRPLabs folder* is provided for the purpose. Add *Channel.gvi* immediately after *Fetch Rx Data* in your receiver. Change the *Fetch RX Data* function to output a cluster instead of waveform so the output data type is compatible with the *Channel.gvi* input. Connect the "Use Channel" and "Propagation Delay" inputs to front panel controls. Connect the "Samples per Symbol" input to the samples per symbol calculated from the actual IQ rate and the symbol rate in your receiver. You may leave the other inputs of *Channel.gvi* unconnected.

Channel.gvi simulates a multipath channel. The default channel parameters create an impulse response given by

$$h(t) = \delta(t) + 0.2\delta(t-\tau) - 0.08\delta(t-2\tau), \quad (17)$$

where τ is the "propagation delay" set by the front panel control. *Channel.gvi* can also add noise to the signal, but we will not be using this feature in this lab project.

Multipath propagation tends to create peaks and dips in the frequency response of the communication channel. This creates the distortion of the received signal that leads to intersymbol interference. Some authors use the so-called *coherence bandwidth* as a measure of the irregularity of the channel frequency response. Roughly, the coherence bandwidth is the frequency interval over which the gain of the channel remains approximately constant. Intersymbol interference will become serious when the signal bandwidth exceeds the coherence bandwidth of the channel. Coherence bandwidth is inversely related to the spread in propagation delays between the first-arriving and last-arriving signal. For the default channel of Eq. (17), the coherence bandwidth is inversely proportional to the parameter τ. The relation is

$$B_{coh} = \frac{0.816}{\tau}, \quad (9.1)$$

where B_{coh} is the coherence bandwidth in Hz when τ is in seconds.

Questions

1. Use Eq. (15) to find E_b/N_0 for a probability of error of $P_e = 10^{-6}$. Now use Eq. (16) to find P_e for an eye opening of $\eta = 0.8$. Repeat for $\eta = 0.5$. Using the same E_b/N_0, find the value of eye opening η that will give $P_e = 10^{-5}$.

2. Suppose $g(0) = 1$, $g(T) = 0.2$, $g(2T) = -0.08$, and $g(nT) = 0$ otherwise. Use Eqs. (9) and (10) to find V_w and V_b. Then use Eq. (14) to find the eye opening.

3. Using Eq. (18), find the coherence bandwidth for $\tau = 50\ \mu s$, $\tau = 100\ \mu s$, and $\tau = 150\ \mu s$.

10.4 Lab Procedure

1. Connect a loopback cable and attenuator between the TX 1 and RX 2 connectors of the USRP. Connect the USRP to your computer and plug in the power to the USRP. Run LabVIEW and open the transmitter that you created in the prelab.

2. Ensure that the transmitter is set up to use

   Carrier Frequency: 915.0 MHz

   IQ Rate: 200 kHz. Note: This sets the value of $1/T_x$.

   Gain: 0 dB

   Active Antenna: TX1

   Symbol rate: 10,000 symbols/s

   Message Length: 1000 bits

   Pulse shaping filter: Root Raised

   Run the transmitter. Use the large STOP button on the front panel to stop transmission connectors.

3. After running the transmitter, observe the spectrum of the transmitted signal. Measure the "main lobe" bandwidth of the transmitted signal. The baseband signal bandwidth is half of the main lobe bandwidth, counting only the positive-frequency components.

4. Ensure that the receiver is set up to use

Carrier Frequency: 915.0 MHz

IQ Rate: 200 kHz. Note: This sets the value of $1/T_z$. Note that T_z is the same parameter as dt.

Gain: 0 dB

Active Antenna: RX2

Symbol rate: 10,000 symbols/s

Message Length: 1000 bits

Pulse shaping filter: Root Raised

Use Channel: off

5. Run the transmitter, then run the receiver. Once the receiver has acquired its data, you may stop the transmitter. The receiver should show a BER of 0.0 or 1.0. Do not be concerned about a BER of 1.0 in this lab project.
6. Observe the eye diagram created by the program you created in step 1 of the prelab. Measure V_w and V_b and calculate the eye opening. Owing to filtering in the USRP, the eye opening should not be 100%.

   The most effective way to measure V_w and V_b from the eye diagram is to use cursors. Click on the eye diagram graph and in the ribbon, click the Graph Parts button. In the dialog window, click the Cursor Legend enable icon. To create a new cursor, click the New Cursor button in the Cursors dialog next to the Eye diagram.

   To measure V_w and V_b, place the X cursor at a time at which the eye opening is widest. This will be near the center of the time axis. Then use the Y cursor to measure V_w and V_b. Do not move the X position between measurements. You will get a more accurate pair of measurements if you zoom in on the relevant part of the eye diagram before positioning the Y cursor. (Graph Parts button, Graph Tools)

7. Set the propagation delay τ to $50\ \mu$s, set Use Channel to "on," and repeat steps 5 and 6. Repeat for $\tau = 100\ \mu$s and $\tau = 150\ \mu$s. Prepare a table showing the eye opening for each value of τ.

Questions

1. Why in this lab project are we not concerned if the BER turns out to be 1.0?

2. Multipath propagation is common in the cellular telephone environment. Propagation delays can vary depending on the locations of the base station, the mobile unit, and nearby buildings, but these delays do not depend on symbol rate. However, with each new generation of cellular service the symbol rate increases. Based on your findings in this lab project, discuss the relationship between symbol rate, coherence bandwidth, eye opening, and BER. What happens as the symbol rate increases?

10.5 Report

Prelab

Hand in documentation for your modified receiver including the eye diagram. Also include documentation for any functions you created. To obtain documentation, print out legible screenshots of the front panel and block diagram.

Answer all of the questions in the Prelab section marked *Questions*.

Lab

Submit the modified receiver including the eye diagram. Resubmit documentation for any functions you modified during the lab. Submit the table required in Lab Procedure step 7 above. Answer the questions in the Lab Procedure section marked *Questions*.